Example b) Shifted Ellipses p. 7 Ch 11

Putting into the correct form

$$9y^2 - 36y + 24x = -4(x^2 + 9)$$

- Expand the right side so you can gather x terms and y terms on the left and constants on the right

$$9y^2 - 36y + 24x = -4x^2 - 36$$

- Gather x's & y's on left

$$9y^2 - 36y + 4x^2 + 24x = -36$$

Complete the Square

- Complete the square for the y's

  Step 1: Factor out the leading coefficient

$$9(y^2 - 4y \qquad) + 4x^2 + 24x = -36$$

  Step 2: Complete the square

$$\left({}^{1}/_{2} \bullet 4\right)^2 = (2)^2 = 4$$

So, $9(y^2 - 4y + 4) + 4x^2 + 24x = -36 + 36$

Remember 9•4 = 36 was added in the left, not 4!

Rewrite, $9(y - 2)^2 + 4x^2 + 24x = 0$

Finish Completing the Square

- Complete the square for the x's

Step 1: Factor out the leading coefficient

$$9(y-2)^2 + 4(x^2 + 6x \quad) = 0$$

Step 2: Complete the square

$$(^1/_2 \bullet 6)^2 = (3)^2 = 9$$

So, $9(y-2)^2 + 4(x^2 + 6x + 9) = 0 + 36$

Remember 4•9 = 36 was added in the left, not 9!

Rewrite, $9(y-2)^2 + 4(x+3)^2 = 36$

Getting 1 as the Constant

Step 3: Divide all terms by 36 to get constant equal to 1

$$\frac{9(y-2)^2}{36} + \frac{4(x+3)^2}{36} = \frac{36}{36}$$

So,

The Correct Form Is:

$$\frac{(y-2)^2}{4} + \frac{(x+3)^2}{9} = 1$$

Meaning: $a^2 = 9$ & $b^2 = 4$, with a center (-3, 2) & this ellipse has a major axis that is horizontal