Example 2 Ellipses p. 6 Ch 11

$$4x^2 + 25y^2 = 100$$

- We will need this one in the correct form. Start by dividing by 100.

$$\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$$

- Now, simplify

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

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- Get a, b & c
- a^2 is the larger denominator

  so, $a^2 = 25$ so, $a = 5$
- b^2 is the smaller denominator

  so, $b^2 = 4$ so, $b = 2$
- $c^2 = a^2 - b^2$

  so, $c = \sqrt{c^2} = \pm\sqrt{25 - 4} = \pm\sqrt{21} = \approx\pm 4.6$

  so, $c = \approx\pm 4.6$

a) Give the Vertices

- The vertices are $(\pm a, 0)$ since this ellipse has a major axis that is horizontal

$V_1(-5,\ 0)$ & $V_2(5,\ 0)$

b) Find the Foci

- Use c to give the foci. For an ellipse which a vertical major axis (x^2 denominator > y^2 denominator) the foci will be (c, 0) & (-c, 0)

So, $F_1(-\sqrt{21}, 0)$ & $F_2(\sqrt{21}, 0)$

Note: *$\sqrt{21} \approx 4.6$*

c) Find the Eccentricity

- The eccentricity tells us how “squashed” the ellipse is around its major axis. $e = {}^{c}/_{a}$

So, $e = {}^{\sqrt{21}}/_{5} = {}^{\sqrt{21}}/_{5}$

Note: This is looking less like a circle because it is fairly close to 1.

d) Find the Major Axis length

- The major axis is horizontal since the larger denominator is on the x^2. That is a^2 & $a = 5$

Major Axis Length: $2(5) = 10$

So, we see that the vertices being at (-5, 0) & (-5, 0) puts them 10 units apart which is the length of the major axis.

e) Find the Minor Axis length

- The minor axis is vertical since the smaller denominator is on the y^2. That is b^2 & $b = 2$

Minor Axis Length: $2(2) = 4$

So, we see that two points on a horizontal line through the center are at (0, 2) & (0, -2) putting them 4 units apart which is the length of the minor axis.

e) Sketch the graph

- 1st Place the vertices
- 2nd Place the foci
- 3rd Place the 2 points

on the minor axis

- 4th Draw the ellipse