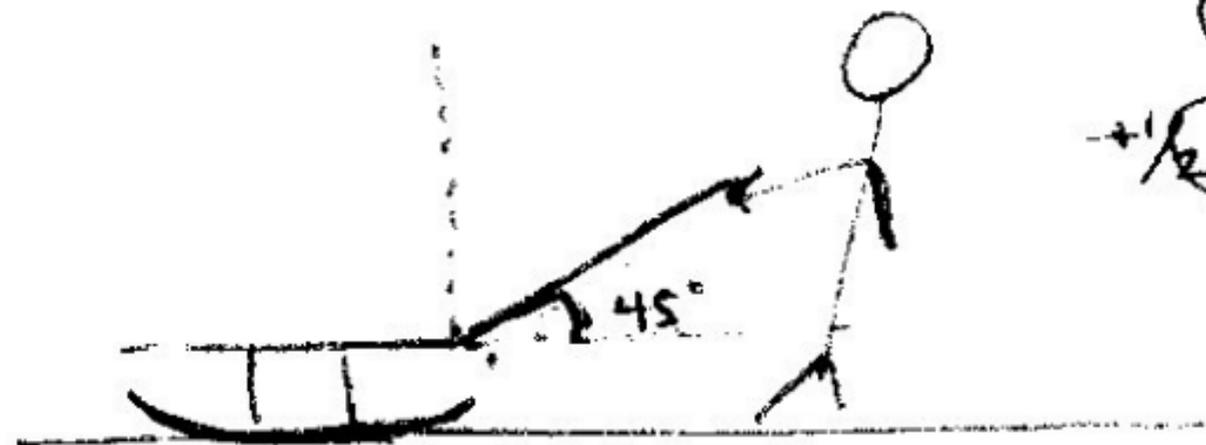


7. A man pulls a sled horizontally by exerting 64 pounds of force on the rope that is tied to its front end. If the rope makes an angle of  $45^\circ$  with the horizontal, find the work done in moving the sled 35 feet (that's a horizontal direction). Round to the nearest  $10^{\text{th}}$  of a foot-pound.



$$F = 64 \cos 45^\circ i + 64 \sin 45^\circ j$$

$$D = 35i + 0j$$

Dot Product  
+1/2

$$W = F \cdot D = (64 \cos 45^\circ)(35) + (64 \sin 45^\circ)(0)$$

$$= (32\sqrt{2})(35)$$

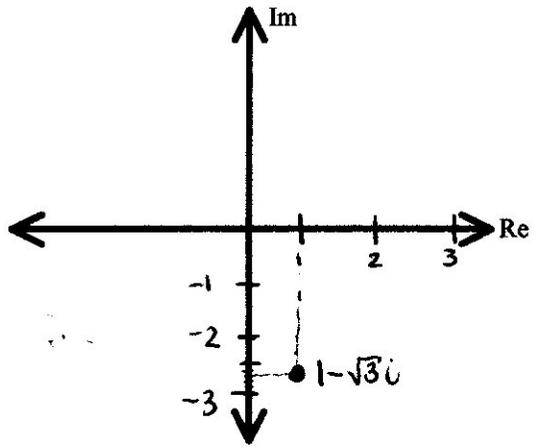
$$\approx 1583.9 \text{ ft} \cdot \text{lbs.}$$

8. For the 3 conics below, put them into the form from which they can be graphed easily (the form which you learned to put each into by completing the square and manipulating

9. For the complex number:  $z = 1 - \sqrt{3}i$

a)  $\frac{11}{11}$

Graph the number in the complex coordinate system shown here



b)  $\frac{12}{12}$

Find the modulus,  $|z|$  of  $z$

$$Z = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

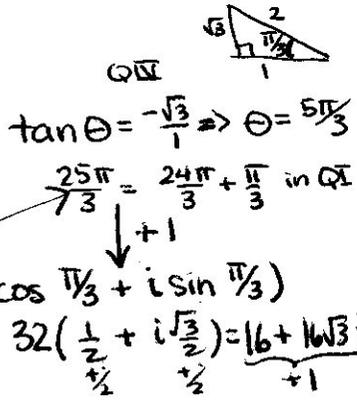
Using de Moivre's theorem

Find the fifth power of the complex number!

c)  $\frac{16}{16}$

\* Simplify completely

$$Z^5 = 2^5 \left( \cos \frac{5 \cdot 5\pi}{3} + i \sin \frac{5 \cdot 5\pi}{3} \right) = 32 \left( \cos \frac{25\pi}{3} + i \sin \frac{25\pi}{3} \right) = 32 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$



\* Leave it

Find the cube roots of the complex number:

$$k = 0, 1, 2$$

+1

$$W_0 = 2^{1/3} \left[ \cos \frac{5\pi/3 + 2(0)\pi}{3} + i \sin \frac{5\pi/3 + 2(0)\pi}{3} \right] = \sqrt[3]{2} \left( \cos \frac{5\pi}{9} + i \sin \frac{5\pi}{9} \right)$$

+1

$$W_1 = 2^{1/3} \left[ \cos \frac{5\pi/3 + 2(1)\pi}{3} + i \sin \frac{5\pi/3 + 2(1)\pi}{3} \right] = \sqrt[3]{2} \left( \cos \frac{11\pi}{9} + i \sin \frac{11\pi}{9} \right)$$

+1

$$W_2 = 2^{1/3} \left[ \cos \frac{5\pi/3 + 2(2)\pi}{3} + i \sin \frac{5\pi/3 + 2(2)\pi}{3} \right] = \sqrt[3]{2} \left( \cos \frac{17\pi}{9} + i \sin \frac{17\pi}{9} \right)$$

At least simplify

\*10. Solve the following  $z^3 + \sqrt{3}i - 1 = 0$  Give approximate values. Don't work harder than you have to!

+1  $z^3 = 1 - \sqrt{3}i$

$z = \text{cube root of } 1 - \sqrt{3}i$  (see #9)

Check to see if they referenced the answers to last part of last problem +1

11. Write  $r = 6 \cos \theta$  in rectangular form. For full credit complete the square. I will give you some options for the final rectangular form, but remember this is not multiple choice!

- a)  $(x-3)^2 + y^2 = 9$
- b)  $x^2 + y^2 = 6$
- c)  $(x-6)^2 + (y-3)^2 = 9$
- d)  $(x-6)^2 + y^2 = 3$

Multiply by  $r$  +1

$$r \cdot (r = 6 \cos \theta) \Rightarrow r^2 = 6r \cos \theta \Rightarrow x^2 + y^2 = 6x$$

$(\frac{1}{2} \cdot 6)^2 = 3^2 = 9$

$$\Rightarrow x^2 - 6x + y^2 = 0 \Rightarrow \boxed{(x-3)^2 + y^2 = 9}$$

Complete the square +1/2

12. Write  $x^2 + (y-1)^2 = 1$  in polar form. For full credit solve completely for  $r$ . I will give you some options for the final rectangular form, but remember this is not multiple choice!

- a)  $r = 2 \sin \theta$
- b)  $r = 2 \cos \theta$
- c)  $r^2 = 1$
- d)  $r = 2 + \cos \theta$

$$x^2 + \underbrace{y^2 - 2y + 1}_{+1/2} = 1 \Rightarrow \underbrace{x^2 + y^2}_{+1/2} = 2y \Rightarrow \frac{r^2}{r} = \frac{2r \sin \theta}{r}$$

$$\Rightarrow \boxed{r = 2 \sin \theta} +1/2$$

Note:  $(r \cos \theta)^2 + (r \sin \theta - 1)^2 = 1$  also works if done correctly.

§8.2

#7 p. 594 Ed 5

#19 p. 553 Ed 6

$$r = 2 - \sin \theta$$

Polar Axis:  $2 - \sin(-\theta) = 2 + \sin(\theta) \neq r$

∴ this is not symmetric to polar axis

Pole:  $2 - \sin(\theta + \pi) = 2 - (\sin \pi \cos \theta + \cos \pi \sin \theta)$   
 $= 2 - (-\sin \theta) \neq r$

∴ This is not symmetric to pole

Line  $\theta = \frac{\pi}{2}$ :  $2 - \sin(\pi - \theta) = 2 - (\sin \pi \cos \theta - \sin \theta \cos \pi)$   
 $= 2 - \sin \theta = r$

∴ This is symmetric to  $\theta = \frac{\pi}{2}$

#10 p. 594 Ed 5

#12 p. 553 Ed 6

$$r = 5 \cos \theta \csc \theta$$

Polar Axis:  $5 \cos(-\theta) \csc(\theta) = 5 \cos \theta (-\csc \theta) \neq r$

∴ This is not symmetric to polar axis

Pole:  $5 \cos(\theta + \pi) \csc(\theta + \pi) = 5 \cos(\theta + \pi) \cdot \frac{1}{\sin(\theta + \pi)}$   
 $= 5 (\cos \pi \cos \theta - \sin \pi \sin \theta) \cdot \frac{1}{\sin \pi \cos \theta + \sin \theta \cos \pi}$

$$= 5(-\cos \theta) \cdot \frac{1}{-\sin \theta} = 5 \cos \theta \csc \theta = r$$

∴ This is symmetric to pole

Line  $\theta = \frac{\pi}{2}$ :  $5 \cos(\theta - \pi) \csc(\theta - \pi) = \frac{5 \cos(\theta - \pi)}{\sin(\theta - \pi)}$

$$= \frac{5(\cos \pi \cos \theta + \sin \theta \sin \pi)}{\sin \pi \cos \theta - \cos \pi \sin \theta} = \frac{5(-\cos \theta)}{\sin \theta}$$

$$= -5 \cos \theta \csc \theta \neq r$$

∴ This is not symmetric to  $\theta = \frac{\pi}{2}$

# Symmetry for $\sin 2\theta$

$$\sin 2\theta = 2 \sin(\theta + \pi) \cos(\theta + \pi)$$

$$= 2(\sin \theta \cos \pi + \sin \pi \cos \theta)(\cos \theta \cos \pi - \sin \theta \sin \pi)$$

$$= 2 \sin \theta \cos \theta$$

$$2(\sin -\theta)(\cos -\theta)$$

$$2(-\sin \theta)(\cos \theta)$$

which is opposite of y orig. so not symmetric to  ~~$\pi/2$~~  polar

Y. Butterworth

Test #2a - M22 F11

Page 2 of 2 over →

Not symmetric to  $\pi/2$  either

$$\sin 2\theta = 2 \sin(\pi - \theta)(\cos(\pi - \theta))$$

$$2[\sin \pi \cos \theta - \sin \theta \cos \pi][\cos \pi \cos \theta + \sin \pi \sin \theta] = 2 \sin \theta \cos \theta$$

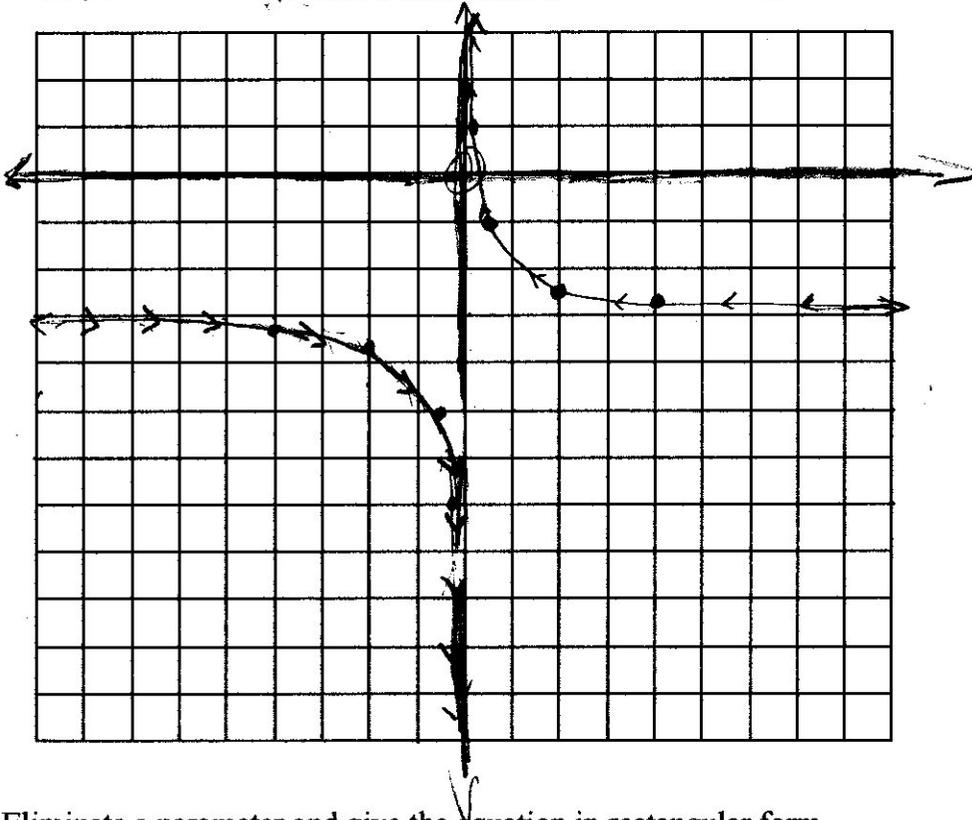
(+2)

12. Graph the parametric equation:  $x = \frac{1}{t}$  &  $y = t - 3$  on  $(-\infty, \infty)$

a) Fill in the following table 1<sup>st</sup>

t	$X = \frac{1}{t}$	$Y = t - 3$
$\frac{1}{4}$	4	$-2\frac{3}{4} = -\frac{11}{4}$
$\frac{1}{2}$	2	$-2\frac{1}{2} = -\frac{5}{2}$
2	$\frac{1}{2}$	-1
4	$\frac{1}{4}$	1
$-\frac{1}{4}$	-4	$-3\frac{3}{4} = -\frac{13}{4}$
$-\frac{1}{2}$	-2	$-3\frac{1}{2} = -\frac{7}{2}$
-2	$-\frac{1}{2}$	-5
-4	$-\frac{1}{4}$	-7

Fill in  
+2



Shape +1

EC Eliminate a parameter and give the equation in rectangular form.

$$x t = 1 \Rightarrow t = \frac{1}{x} \quad \text{Solve for variable } t$$

$$y = \frac{1}{x} - 3$$

Substitute into other eq.  
+1

8. For the 3 conics below, put them into the form from which they can be graphed easily (the form which we learned to put each into by completing the square and manipulating the equation). Then indicate if the conic is a hyperbola, an ellipse or a parabola.

*italics*

a)  $\frac{1}{2}$

$$\frac{16y^2}{64} - \frac{6x^2}{64} = \frac{64}{64} \Rightarrow$$

$$+1 \cdot \frac{y^2}{4} - \frac{x^2}{\frac{64}{6}} = 1$$

Hyperbola  $+\frac{1}{2}$

b)  $\frac{1}{2}$

$$4x^2 + 9y^2 - 16x - 20 = 0$$

$$(4x^2 - 16x) + 9y^2 = 20$$

$$\frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$$

Ellipse

c)  $\frac{1}{2}$

$$y^2 + 2y - 12x = -37$$

$$y^2 + 2y + 1 = 12x - 37 + 1$$

$$(y+1)^2 = 12(x-3)$$

Parabola

9. Give the following for the parabola:

$$x^2 + 2y = 0$$

$$x^2 = -2y$$

$$4p = -2$$

$$p = -\frac{1}{2}$$

a)  $\frac{1}{1}$

The vertex  $(0, 0)$

b)  $\frac{1}{1}$

The focus  $(0, -\frac{1}{2})$

If they match it's OK only take off  $\frac{1}{2}$  pt.

c)  $\frac{1}{1}$

The length of the latus rectum  $|4 \cdot -\frac{1}{2}| = |-2| = 2$

d)  $\frac{1}{1}$

Two points on parabola, that are on the same horizontal line as the focus  $(-1, -\frac{1}{2})$  &  $(1, -\frac{1}{2})$

e)  $\frac{1}{1}$

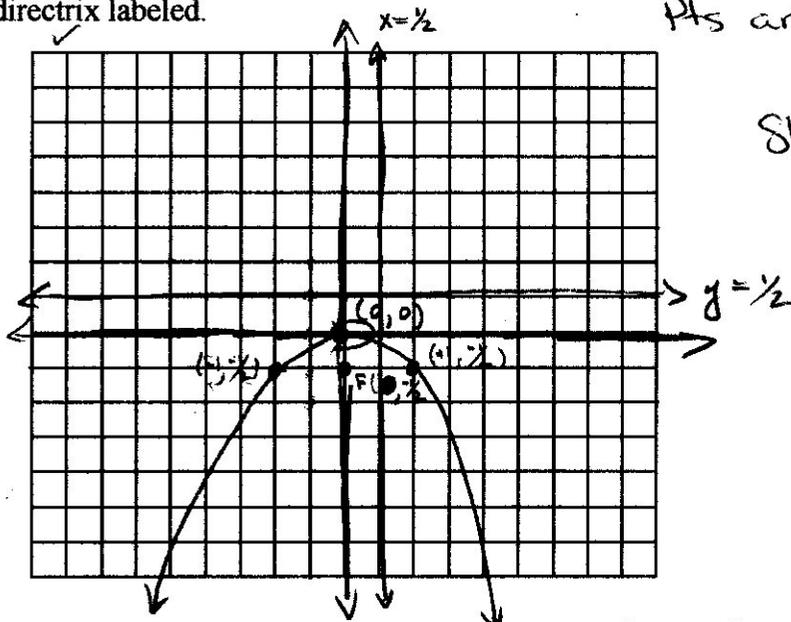
The directrix  $y = \frac{1}{2}$ . Must be an equation of a line.

f)  $\frac{1}{1}$

The equation of the line of symmetry

$$x = 0$$

- g) Graph the parabola with the vertex, focus, points on horizontal with the focus and the directrix labeled.



Pts are there

Shape +  $\frac{1}{2}$

10. Find the following for the hyperbola:

$$\frac{(x+1)^2}{16} - \frac{y^2}{9} = 1$$

The center

$$(-1, 0)$$

The foci

$$(-6, 0) \text{ \& } (4, 0)$$

The asymptotes

$$y - 0 = -\frac{3}{4}(x - (-1)) \Rightarrow y = -\frac{3}{4}x - \frac{3}{4}$$

$$y - 0 = \frac{3}{4}(x - (-1)) \Rightarrow y = \frac{3}{4}x + \frac{3}{4}$$

$$c^2 = b^2 + a^2$$

$$c = \sqrt{16 + 9} = 5$$

11. Find the following for the ellipse:

$$\frac{(y-1)^2}{100} + \frac{x^2}{25} = 1$$

$b=10$        $a=5$

The vertices

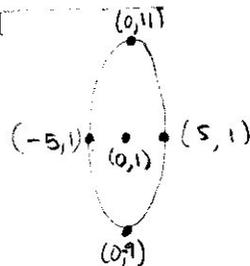
$$(0, 11) \text{ \& } (0, -9)$$

The center

$$(0, 1)$$

The points on the ellipse on the minor axis

$$(-5, 1) \text{ \& } (5, 1)$$



- (a) +
- (b) +
- (c) +
- (d) +

# Means from Class

$10, m_1, m_2, 18 \Rightarrow$  Find  $m_1$  &  $m_2$

$$18 = 10 + d(4 - 1)$$

$$\frac{8}{-10} = \frac{3d}{-10}$$

$$d = \frac{8}{3} = 2\frac{2}{3}$$

$$\therefore \begin{cases} m_1 = 10 + 2\frac{2}{3} = 12\frac{2}{3} \\ m_2 = \underbrace{10 + 2\frac{2}{3}}_{12\frac{2}{3}} + 2\frac{2}{3} = 15\frac{1}{3} \end{cases}$$

$$\text{or } m_2 = 10 + \frac{8}{3}(3-1)$$

$$= 10 + \frac{8}{3} \cdot 2 \Rightarrow \frac{30}{3} + \frac{16}{3} = \frac{46}{3} = 15\frac{1}{3}$$

$10, m_1, m_2, m_3, 18 \Rightarrow$  Find  $m_1, m_2$  &  $m_3$

$$18 = 10 + d(5-1)$$

$$8 = 4d \Rightarrow d = 2$$

$$m_1 = 10 + 2 = 12$$

$$m_2 = 12 + 2 = 14$$

$$m_3 = 14 + 2 = 16$$

$\$11.2$   
 $\$3$  ES  
 $\$67$  EB  
 $\$12.2$

k days of x-mas

1st day = 1

2nd day = 1 + 2

3rd day = 1 + 2 + 3

4th day = 1 + 2 + 3 + 4

∴ Sequence is arithmetic w/  $a=1$  &  $d=1$

so,  $a=1$  &  $a_n=12$

so  $S_{12} = 12 \left( \frac{1+12}{2} \right)$   
 $= \frac{12}{1} \cdot \frac{13}{2} = \boxed{78}$

The Interest Problem is Bogus!! Sorry.

$\$11.6$  p. 869 26 ES  
 $\$12.6$  p. 829 30 EB

1st four terms of  $(x^{1/2} + 1)^{30}$

$1^{st} \binom{30}{0} (x^{1/2})^{30} = x^{15}$   
 $2^{nd} \binom{30}{1} (x^{1/2})^{29} (1)^1 = 30x^{29/2}$   
 $3^{rd} \binom{30}{2} (x^{1/2})^{28} (1)^2 = 435x^{14}$   
 $4^{th} \binom{30}{3} (x^{1/2})^{27} (1)^3 = 4060x^{27/2}$

Extra one  
 # 34 Ed 5

2nd term of  $(x^2 - \frac{1}{x})^{25}$

$\binom{25}{1} (x^2)^{24} \left(-\frac{1}{x}\right)^1 = 25x^{48} \cdot \frac{-1}{x} = \boxed{-25x^{47}}$

\* Remember 2nd is "n-1"th

# Fraction as a Decimal

$$\textcircled{2} \quad \underline{2.1} \underline{125} \underline{125} \dots$$

$$\frac{21}{10} + \frac{125}{10,000} + \frac{125}{10,000,000} + \dots$$

From place values ...

$$a = \frac{125}{10,000} \quad \text{since} \quad \frac{125}{10,000,000} = a \left( \frac{1}{1000} \right)^{2-1} \Rightarrow \cancel{a} a$$

$$r = \left( \frac{1}{1000} \right)$$

## Infinite Sum of Geometric Sequence

$$S = \frac{a}{1-r} = \frac{\frac{125}{10,000}}{1 - \frac{1}{1000}} = \frac{\frac{125}{10000}}{\frac{999}{1000}} = \frac{125}{10000} \cdot \frac{1000}{999}$$

$$= \frac{125}{9990} \quad \text{Which represents } 0.0125125\dots$$

Then add into  $\frac{21}{10}$

$$\frac{21}{10} + \frac{125}{9990} = \dots$$

8812.5

#5

Ed6 p.819

$$5 + 8 + 11 + \dots + (3n+2) = \frac{n(3n+7)}{2}$$

$$P(1) = \frac{1(3(1)+7)}{2} = \frac{10}{2} = 5$$

∴ Assume

$$P(k) \text{ is true so } P(k) = \frac{k(3k+7)}{2}$$

So starting w/ left

$$5 + 8 + 11 + \dots + 3k + 2 + 3(k+1) + 2$$

By hypothesis

$$\rightarrow \frac{k(3k+7)}{2} + 3(k+1) + 2$$

$$= \frac{3k^2 + 7k}{2} + \frac{6k + 10}{2}$$

$$= \frac{3k^2 + 13k + 10}{2} = \frac{(3k+10)(k+1)}{2}$$

$$= \frac{[3(k+1)+7](k+1)}{2}$$

∴  $P(k+1)$   
follows from  
 $P(k)$